DTIC FILE COPY

(2)

AD-A216 468

REPORT DOCUMENTATION PAGE

DTIC ELECTE JAN 05 1990 S D D Cg

1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY	
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	
3. DISTRIBUTION / AVAILABILITY OF REPORT	Approved for public release; distribution unlimited.
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	
5. MONITORING ORGANIZATION REPORT NUMBER(S)	AFOSR-TR- 89-1895
6a. NAME OF PERFORMING ORGANIZATION	Virginia Polytechnic Institute and State University
6b. OFFICE SYMBOL (If applicable)	
7a. NAME OF MONITORING ORGANIZATION	AFOSR/NM
6c. ADDRESS (City, State, and ZIP Code)	Blacksburg, Virginia 24061
7b. ADDRESS (City, State, and ZIP Code)	AFOSR/NM Bldg 410 Bolling AFB DC 20332-6448
8a. NAME OF FUNDING / SPONSORING ORGANIZATION	AFOSR
8b. OFFICE SYMBOL (If applicable)	NM
9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	AFOSR 86-0085
8c. ADDRESS (City, State, and ZIP Code)	AFOSR/NM Bldg 410 Bolling AFB DC 20332-6448

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
61102F	2304	A1	

11. TITLE (Include Security Classification)

Final Technical Report: Energy Decay and Bounday Control for Distributed Parameter Systems with Viscoelastic Damping

12. PERSONAL AUTHOR(S)

Hannsgen, Kenneth Bruce and Wheeler, Robert Lee

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final	FROM 86/06/01 TO 89/05/31	89/07/24	17

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This report concerns several aspects of active and passive stabilization of materials and structures, together with related dynamic problems with potential applications to stabilization. Results are described for determination of the interacting effects of viscoelastic and feedback dissipation in the damping of oscillations in certain rods and beams. Loss of stability or of well-posedness due to feedback delays is described. Additional results concern analysis of energy decay in elastic beams and an investigation of the Signorini problem for motion of an elastic body that abuts a rigid obstacle. Numerical and analytic studies are described for Bingham fluids as well as for several problems involving two-fluid flows. Finally, work on control problems for the Navier-Stokes equations is summarized.

90 01 04 065

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	Unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code)	22c. OFFICE SYMBOL
Lt. Col. James M. Crowley	(202) 767-5025	NM

FINAL TECHNICAL REPORT

ENERGY DECAY AND BOUNDARY CONTROL FOR DISTRIBUTED PARAMETER SYSTEMS WITH VISCOELASTIC DAMPING

AFOSR GRANT AFOSR-86-0085

for the period

1 June 1986 - 31 May 1989

by

K. B. Hannsgen and R. L. Wheeler

Department of Mathematics

Interdisciplinary Center for Applied Mathematics

Virginia Polytechnic Institute and State University

Blacksburg, Virginia 24061-0123

I. SUMMARY

The subject grant supported research on dynamic behavior of viscoelastic structures, with emphasis on the interaction between passive viscoelastic damping and active feedback damping. A general investigation of the feasibility of stabilizing torsional vibrations in a viscoelastic rod, with a class of velocity-stress feedback schemes, was completed. An analytic study of bending waves in a beam, with particular attention to severe loss of stability due to feedback delays as well as contrasts between different models for the beam, was made and reported. Sharp necessary and sufficient spectral criteria for loss of well-posedness through feedback delays in a wide class of abstract differential equations were obtained. Related analytic and numerical work on certain two-fluid problems, with potential applications to viscoelasticity, was carried out. Studies were made of energy decay in the dynamic Signorini problem (involving an elastic bar vibrating against a rigid obstacle) and for a nonhomogeneous bar with locally distributed damping. Theoretical and numerical results were obtained for control problems involving the Navier-Stokes equations, as well as for the evolution inequality governing a Bingham fluid.

II. INTRODUCTION

This report contains a summary of the results partially supported under the Air Force Office of Scientific Research Grant AFOSR-86-0085 during the period June 1, 1986 to May 31, 1989. This project was concerned with the interaction between mechanical (viscoelastic) dissipation and active feedback control mechanisms in the decay of energy in various bodies and structures. The principal investigators were Professors Kenneth B. Hannsgen and Robert L. Wheeler.

During the three year period, 29 research papers were completed. Also, 7 faculty (2 principal investigators, 2 senior investigators, 3 post docs) and 7 graduate students were partially supported by the grant. One of these students has received his Ph.D. degree, and the remaining students are expected to complete their work in the near future.

We have investigated many aspects of active and passive stabilization of materials and structures, together with related dynamic problems with potential applications to such stabilization problems. Specifically, we

1.) determined the interacting effects of viscoelastic dissipation and feedback dissipation in the damping of oscillations in certain viscoelastic rods and beams, and in simple hybrid systems containing viscoelastic rods or beams [6, 7, 19, 20].

2.) determined how feedback delays can cause a loss of stability or even a loss of well-posedness and how these effects depend on viscoelastic moduli and the nature of the mechanical model [1, 3, 7, 18].

3.) established that energy decays at an exponential rate in the elastic Timoshenko beam with a boundary control mechanism and in longitudinal or torsional vibrations of a nonhomogeneous bar composed of two segments, with distributed damping applied to just one of the segments [2, 23].

4.) provided the first mathematical analysis of the dynamic Signorini problem, involving an elastic or viscoelastic body constrained by a rigid obstacle [21, 22].

The above work is closest to the immediate aims of the project. In addition, we developed a numerical solution method, together with its theoretical foundation, for the evolution variational

inequality for a rigid-visco-plastic (Bingham) fluid [4, 11-13]. Other work involved numerical and theoretical analysis of stability and instability in two-fluid problems [8, 10, 24]; optimization and control problems for the Navier-Stokes equations [26-29]; and the preservation of controllability under approximation for various finite element and finite difference approximations of parabolic systems [5].

III. HIGHLIGHTS OF RESEARCH

In this section we discuss some of the work performed under the grant. Reprints of the papers will be forwarded to AFOSR as soon as they become available.

(1) Damping of oscillations in viscoelastic rods and beams.

The study of torsional vibrations in a viscoelastic rod has involved Hannsgen and Wheeler, together with Yuriko Renardy and Wolfgang Desch. Aspects of this work are reported in [1, 6, 7, 20] and have been presented at several scientific meetings.

A linear Boltzmann model of rate type, involving the stress relaxation function $A(t)$, is used. In practice, one can assume that $A(t)$ is completely monotone on $(0, \infty)$, although this is not always necessary mathematically. For viscoelastic solids, we have $E \equiv A(\infty) > 0$; the behavior of $A(t)$ for small t reflects important physical properties of the material. Indeed, a principal achievement of this project was a detailed examination of the relation between small t behavior of $A(t)$ and the effectiveness and robustness (with respect to time delays; see (2) below) of boundary feedback stabilization methods.

Results for the problem of torsional vibrations included a theoretical and numerical description of the characteristic spectrum and an existence-uniqueness result based on inversion of Laplace transforms. These results covered the case of a constant gain parameter relating boundary velocity to the stress feedback. The analysis included the case of pure boundary feedback as well as the case of feedback through an inertial mass (a simple hybrid system). In addition, in the hybrid system case an input-output problem was formulated for variable gain feedbacks [20], as a step towards implementable schemes that realize, with arbitrarily small error, the theoretically best possible exponential rates of energy decay.

Most recently, Hannsgen and Wheeler have examined the special case where $A(t) - E$ does not decay exponentially at infinity so that energy cannot be expected to decay exponentially, even with boundary feedback. While this case is rare in practice, it permits one to analyze in detail the decay rates and sensitivity to feedback of so-called creep modes that are always present in

viscoelastic models. The separation of motion into creep modes and oscillatory modes appears to be of fundamental importance in stabilization and control problems involving viscoelastic damping. Preliminary results were presented at the SIAM Conference on Control in the 90's in May, 1989.

Much of the behavior for rods carries over to bending vibrations in viscoelastic beams. Work reported in [3, 19] focused on certain new phenomena, such as dependence on the model (Euler-Bernoulli vs. Timoshenko) and the decoupling of shear and bending oscillations at high frequencies.

(2) Effect of feedback delays.

It has been known for some time that even the smallest delays in a boundary feedback scheme can destabilize, rather than stabilize, vibrations. The studies cited in Section (1) above include an analysis of this phenomenon for viscoelastic materials and, in particular, a demonstration that materials for which $A(0+) = \infty$ (including, as an important special case, fractional derivative models) are the least susceptible to such destabilization.

In an Euler-Bernoulli beam, destabilization can be so severe that well-posedness is lost. Desch and Wheeler [18] studied this extreme behavior for a very general class of linear systems and gave a simple, sharp criterion that is necessary and sufficient for this behavior to occur. In particular, such a loss of well-posedness due to feedback delay can occur only if the corresponding open loop system is ill-posed in a local L^2 input L^2 output sense. Moreover, loss of well-posedness cannot occur if the distance between successive eigenvalues in the corresponding open-loop system is bounded, as it is for hyperbolic equations.

(3) Energy methods.

Jong Kim and Yuriko Renardy [2] analyzed flexural oscillations of the elastic Timoshenko beam in the context of boundary control. The Timoshenko model, which incorporates the effects of shear flexibility and rotary inertia, is analytically complex. By devising suitable energy functionals associated with the natural energy of the beam, these authors were able to show that a boundary control mechanism can be used to achieve uniform exponential stabilization of the beam. They

also carried out numerical experiments on the spectrum of the Timoshenko beam with boundary control.

In [23], Kim has shown that locally distributed damping can stabilize the motion of a one-dimensional nonhomogeneous elastic medium exponentially fast. Specifically, either longitudinal or torsional vibrations of a nonhomogeneous bar were considered where the bar is composed of two different segments, only one of which is damped. Various combinations of several different segments can be treated by the same method.

(4) Dynamic contact problems.

In [21, 22] Kim has investigated the dynamic Signorini problem. The *Signorini problem* involves contact boundary conditions that model the situation in which the boundary of an elastic body abuts a rigid obstacle so that stress can be applied to the body in only one direction. Mathematically this is a difficult to analyze one-sided nonlinear boundary condition. While the stationary Signorini problem has been fully investigated by many people, the dynamic problem has only been treated formally in some engineering works, and Kim is the first to analyze this problem in a mathematically rigorous fashion. The case of one dimensional elastic and viscoelastic bars was discussed in [21]. The main tool for analysis was a multiplier technique. While this method does not carry over to the higher dimensional case, the result was extended to a higher dimensional wave equation with the aid of the method of compensated compactness [22].

(5) Analysis and finite element approximation of dynamics of Bingham fluids.

A Bingham fluid is a rigid-visco-plastic fluid whose motion is described by an evolution variational inequality. This involves a nondifferentiable functional which causes serious mathematical difficulties in addition to those associated with the Navier-Stokes equations. Using the L^p theory of the Stokes operator, Kim has established the existence of strong solutions in a three dimensional domain [4]; this has been an open question since the fundamental work of Duvaut and Lions in the early 70's. He further extended the result by means of a discretized version of the variation of

constants formula in [12]. These results constitute a foundation for the numerical approximation developed in [13] where solutions are approximated by a backward Euler scheme in the time variable and conforming piecewise linear finite elements augmented by the penalty method. Convergence rates and error estimates are obtained in this paper.

(6) Stability and instability of two-fluid flow problems.

Flows involving two immiscible fluids exhibit phenomena which are totally unexpected from a knowledge of one-fluid flows. While partially supported by funds from this grant, Yuriko Renardy used physical modeling, analytical and computational techniques to study the stability of various steady arrangements in two-fluid flows, and the appearance of new arrangements from unstable ones [8, 10, 24].

A two-layer Couette flow composed of fluids of the upper-convected Maxwell model is analyzed in [8]. It is shown that at low speeds the interfacial mode may become unstable, while other modes stay stable. The shortwave asymptotics of the interfacial mode is analyzed, and a numerical study of the entire spectrum is used to verify the results of this asymptotic analysis.

In joint work [10], M. Renardy and Y. Renardy use bifurcation theory methods to study the oscillatory onset of convection in the two-fluid Bénard problem. Since there are a large number of qualitatively different types of bifurcating solutions, and since the eigenfunctions at criticality are not explicitly known, a combination of analysis and numerical computations is used to investigate this problem.

In [24] Y. Renardy used bifurcation theory techniques to investigate the stability of plane two-layer Couette-Poiseuille flow. The center manifold theorem is used to justify the derivation of the final amplitude evolution equation. The derivation and the numerical results obtained are compared with those of a formal approach, employing multiple scales, that has been used on related problems.

(7) Optimization and control problems for the Navier-Stokes equations.

During his tenure as a Postdoctoral Research Associate supported by the subject grant, Thomas Svobodny began a theoretical and numerical study of optimization and control problems involving the Navier-Stokes equations. The completed work to date was done jointly with Drs. Max Gunzberger and L. Hou [26-29].

The problem of minimizing the L^2 norm of the strain rate deformation tensor with velocity control acting on part of the boundary, where the control is constrained to lie in a closed, possibly bounded, set was studied in [27]. It is shown that a minimum principle follows from the Fredholm structure of the stationary Navier-Stokes equations. The results obtained are applied to a problem of reducing drag on a body immersed in a steady flow by applying blowing and suction on portions of the boundary surface.

Paper [28] deals with the problem of analyzing and approximating the optimality system for finite time regulator problems for the stationary and nonstationary Navier-Stokes equations, without constraints on the velocity control vectors. Paper [29] (some preliminary results are also given in [26]) deals with the more complicated problem of analyzing and approximating the optimality system for the case of Dirichlet boundary control. The regularity of the optimal solution is analyzed and optimal error estimates for the approximation are then derived using a Lagrange multiplier technique.

IV. CUMULATIVE LIST OF PUBLICATIONS

AFOSR Grant AFOSR-86-0085

1. K. B. Hannsgen and R. L. Wheeler, Time delays and boundary feedback stabilization in one-dimensional viscoelasticity, Proc., Third International Conference on Distributed Parameter Systems, F. Kappel, K. Kunish and W. Schappacher eds., Springer Lecture Notes in Control and Information Sciences, Vol. 102, 1987, 136-152.

2. J. U. Kim and Y. Renardy, Boundary control of the Timoshenko beam, SIAM J. Control and Optimization, 25 (1987), 1417-1429.

3. W. Desch, K. B. Hannsgen, Y. Renardy and R. L. Wheeler, Boundary stabilization of an Euler-Bernoulli beam with viscoelastic damping, Proc. 26th IEEE Conference on Decision and Control, Los Angeles, CA 1987, 1792-1795.

4. J. U. Kim, On the initial-boundary value problem for a Bingham fluid in a three dimensional domain, Trans. Amer. Math. Soc., 204 (1987), 751-770.

5. J. A. Burns and G. H. Peichl, On robustness of controllability for finite dimensional approximations of distributed parameter systems, Proc. IMACS/IFAC-DP'87, Hiroshima, Japan, 1987, 491-496. Also in IMACS Transactions on Scientific Computing, to appear.

6. K. B. Hannsgen and R. L. Wheeler, Existence and decay estimates for boundary feedback stabilization of torsional vibrations in a viscoelastic rod, Proc. C.I.R.M. Meeting on Volterra Integrodifferential Equations in Banach Spaces and Applications, Trento, Italy, 1987, Pitman, to appear.

7. K. B. Hannsgen, Y. Renardy and R. L. Wheeler, Effectiveness and robustness with respect to time delays of boundary feedback stabilization in one-dimensional viscoelasticity, SIAM J. on Control and Optimization, 26 (1988), 1200-1234.

8. Y. Renardy, Stability of the interface in two-layer Couette flow of upper convected Maxwell liquids, J. Non-Newtonian Fluid Mech. 28 (1988), 88-115.

9. W. Desch and R. Grimmer, Singular relaxation moduli and smoothing in three-dimensional viscoelasticity, Trans. Amer. Math. Soc., to appear.

10. M. Renardy and Y. Renardy, Bifurcating solutions at the onset of convection in the Bénard problem for two fluids, Physica D, 32 (1988), 227-252.

11. J. U. Kim, Regularity of the motion of a Bingham fluid, Proc. International Conference on Theory and Applications of Differential Equations, Columbus, Ohio, 1988, to appear.

12. J. U. Kim, Semi-discretization method for three dimensional motion of a Bingham fluid, SIAM J. Math. Anal., to appear.

13. J. U. Kim, A finite element approximation of three dimensional motion of a Bingham fluid, Math. Model. and Num. Analysis, 23 (1989), 293-333.

14. K. Kunisch and G. H. Peichl, On the shape of the solutions of second order parabolic partial differential equations, Journal of Differential Equations, 75 (1988), 329-353.

15. A. Al-Droubi, A two dimensional parabolic-elliptic interface problem, submitted to Quart. Appl. Math.

16. W. Desch and R. K. Miller, Exponential Stabilization of Volterra integral equations with singular kernels, J. Integral Equations Appl., to appear.

17. A. Al-Droubi and M. Renardy, Energy methods for a parabolic-hyperbolic interface problem arising in electromagnetism, Z. Angew. Math. Phys., 39 (1988), 931-936.

18. W. Desch and R. L. Wheeler, Destabilization due to delay in one-dimensional feedback, Proc. Fourth International Conference on Distributed Systems, Vorau, Austria, 1988, to appear.

19. K. B. Hannsgen, Stabilization of the viscoelastic Timoshenko beam, Proc. Fourth International Conference on Distributed Systems, Vorau, Austria, 1988, to appear.

20. W. Desch, K. B. Hannsgen and R. L. Wheeler, Feedback stabilization of a viscoelastic rod, Proc. ComCon 88, Advances in Communications and Control Systems, Baton Rouge, LA, 1988, 1271-1277. Also to appear in Springer-Verlag Volume on Advances in Computing and Controls.

21. J. U. Kim, One-dimensional dynamic contact problem in linear visco-elasticity, submitted.

22. J. U. Kim, A boundary thin obstacle problem for a wave equation, Comm. Partial Diff. Equations, to appear.

23. J. U. Kim, Exponential decay of the energy of a one-dimensional nonhomogeneous medium, submitted.

24. Y. Renardy, A Couette-Poiseuille flow of two fluids in a channel, Physics of Fluids, to appear.

25. T. Svobodny, Stability of nonlinear observers for dissipative ODEs, International J. Control, to appear.

26. M. Gunzburger, L. Hou, and T. Svobodny, Numerical approximation of an optimal control problem associated with the Navier-Stokes equations, Applied Math. Lett. 2 (1989), 29-31.

27. M. Gunzburger, L. Hou, and T. Svobodny, Boundary velocity control of incompressible flow with an application to viscous drag reduction, submitted.

28. M. Gunzburger, L. Hou, and T. Svobodny, Analysis and finite element approximation of optimal control problems for the Navier-Stokes system with distributed and Neumann controls, submitted.

29. M. Gunzburger, L. Hou, and T. Svobodny, Optimization and approximation problems for the stationary Navier-Stokes system with Dirichlet boundary control, submitted.

V. INVESTIGATORS PARTIALLY SUPPORTED UNDFR THIS GRANT

K. B. Hannsgen - Principal Investigator

R. L. Wheeler - Principal Investigator

J. U. Kim - Senior Investigator

Y. Renardy - Senior Investigator

Gunter Peichl - Postdoctoral Research Associate

Wolfgang Desch - Postdoctoral Research Associate

Akram Al-Droubi - Postdoctoral Research Associate

Thomas P. Svobodny - Postdoctoral Research Associate

R. Miller - Graduate Student (Ph.D. 1988)

D. Hill - Graduate Student (Working on Ph.D.)

Sungkwon Kang - Graduate Student (Working on Ph.D.)

John Ong - Graduate Student (Working on Ph.D.)

Patricia Webb - Graduate Student (Working on Ph.D.)

Scott Inch - Graduate Student (Working on Ph.D.)

Gyou Bong Lee - Graduate Student (Working on Ph.D.)

VI. INTERACTIONS

(a) On-campus activities. The Principal Investigators participated in and organized active seminars in viscoelasticity, functional differential equations and control in the Virginia Tech Mathematics Department. Talks were presented by our Senior Investigators and our Postdoctoral Research Associates and Graduate Students, and by on-campus researchers including Drs. John Burns, Terry Herdman, Gunter Leugering, Michael Renardy, Robert Rogers and David Russell (Mathematics) and Dr. Donald Baird (Chemical Engineering). In addition, a number of visitors to Virginia Tech made presentations in these seminars including

Stuart Antman, University of Maryland
Ronald Bagley, Air Force Institute of Technology
Thomas Bridges, Worcester Polytechnic Institute
Gordon Chen, Texas A & M University
Constantine Dafermos, Brown University
Hans Engler, Georgetown University
Ronald Grimmer, Southern Illinois University
Melvin Heard, University of Illinois at Chicago
William Hrusa, Carnegie-Mellon University
Vilmos Komornik, University of Bordeaux
John Lagnese, Georgetown University
Irena Lasiecka, University of Virginia
Tai-Ping Liu, University of Maryland
Stig-Olof Londen, Helsinki University of Technology
Dahlard Lukes, University of Virginia
Richard MacCamy, Carnegie-Mellon University
Reza Malek-Madani, U.S. Naval Academy
Richard Miller, Iowa State University
Richard Noren, Old Dominion University
Lynn Rogers, Air Force Wright Aeronautical Laboratories
Marshall Slemrod, University of Wisconsin
Roberto Triggiani, University of Virginia
Janos Turi, Worcester Polytechnic Institute
George Weiss, Brown University
Enrike Zuazua, University del Pais Vasco

The visits by Drs. Rogers in 1987 and Bagley in 1989 were part of a series of contacts with researchers at Wright Aeronautical Laboratories and the Air Force Institute of Technology. These interchanges began with visits to Wright-Patterson by Dr. Wheeler in the fall of 1985 and by Drs. Hannsgen and Wheeler in the spring of 1986 and the winter of 1988. These exchanges were

extremely helpful in indicating important research directions for the project through exposure to experimental work in progress at the labs and to the theoretical work, such as that on fractional derivative models.

(b) Talks and presentations

Hannsgen

1. Sixth Annual Southeastern-Atlantic Regional Conference on Differential Equations, October, 1986, (contributed talk).
2. C.I.R.M. meeting on Volterra Integrodifferential Equations in Banach Spaces and Applications, Trento, February, 1987.
3. Southern Illinois University, Carbondale, Illinois, March, 1987, (Seminar).
4. University of Minnesota-Duluth, May, 1987, (Seminar).
5. Fourth International Conference on Control of Distributed Parameter Systems, Vorau, Austria, July, 1988.
6. SIAM Conference on Control in the 90's, San Francisco, May, 1989, (contributed talk).

Wheeler

1. International Conference on Control and Identification of Distributed Systems, Vorau, Austria, July, 1986.
2. Volterra Integrodifferential Equations in Banach Spaces and Applications, Trento, Italy, February, 1987.
3. Helsinki University of Technology, Helsinki, Finland, May 8 and May 12, 1987, (Seminars).
4. Universität Gesamthochschule Paderborn, Paderborn, West Germany, May 25, 1987, (Colloquium).
5. Combined Midwest-Southeast Differential Equations Conference, Vanderbilt University, Nashville, Tennessee, October, 1987.
6. 26th IEEE Conference on Decision and Control, Los Angeles, California, December, 1987.
7. Southern Illinois University, Carbondale, Illinois, March 3 and March 4, 1988, (Colloquium; Seminar).
8. Universität Graz, Graz, Austria, June 23, 1988, (Seminar).

9. Fourth International Conference on Control of Distributed Parameter Systems, Vorau, Austria, July, 1988.
10. Advances in Communications and Control Systems ComCon 88, Baton Rouge, Louisiana, October, 1988.
11. Eighth Southeastern-Atlantic Regional Conference on Differential Equations, Athens, Georgia, November, 1988 (Plenary Lecture).
12. SIAM Conference on Control in the 90's, San Francisco, May, 1989, (invited mini symposium talk).

Kim

1. Georgetown University, Washington, D.C., February, 1988, (Colloquium).
2. International Conference on Differential Equations, Columbus, Ohio, March, 1988.
3. University of Tennessee, Knoxville, September, 1988, (Colloquium).

(c) Other consultations and travel

Dr. Wheeler participated in the UW MIPAC mini workshop organized by UW MIPAC Director Dr. David Russell and held at the UW MIPAC Facility in August of 1986. In addition to attending the series of talks and demonstrations by Dr. Russell on the use of the HP 5451C System Analyzer and other UW MIPAC equipment (partially obtained with AFOSR funds) in several mathematical modeling problems, Wheeler was able to check damping properties of a boron epoxy composite beam and compare the experimental results with theoretical curves predicted by various viscoelastic constitutive relations. This workshop gave Wheeler valuable first hand experience with some of the methods and difficulties encountered in the measurement of damping properties in solids.

Wheeler visited the UW MIPAC Facility again from the 12th to the 14th of August, 1987. Wheeler and Dr. Russell continued their discussions on models for damping and the measurement of damping in solids. Dr. Russell showed Wheeler a new model for damping in beams that he has developed based on thermoelastic considerations. This model leads to the same equations of

motion as the Maxwell-solid viscoelastic model. Dr. Russell also demonstrated some of the new equipment that the MIPAC Facility has obtained since Wheeler's previous visit in August, 1986, including the DISA Laser Vibrometer, and Russell and Wheeler ran some additional experiments on measuring damping in beams with attached lumped masses.

The Principal Investigators have had many fruitful discussions dealing with models for damping in solids with Dr. Russell since he moved to Virginia Tech in the Fall of 1988.

Dr. Hannsgen spent the months of April and May 1987, on Study-Research Leave at the Mathematics Research Center, University of Wisconsin, with travel expenses partially supported by the subject AFOSR funds. While in Madison he discussed various models for damping of beams with UW MIPAC Director Dr. Russell. He also interacted with experts in integro-partial differential equations and continuum mechanics such as Drs. John Nohel and Morton Gurtin. He presented the results in [4] on feedback stabilization of viscoelastic rods at a seminar at MRC. While in the midwest, Hannsgen also gave colloquium lectures on this topic at Southern Illinois University at Carbondale and the University of Minnesota at Duluth. While in Carbondale, he also discussed problems of wave propagation in viscoelastic structures with Dr. Ronald Grimmer of SIU and with Dr. Wilhelm Schappacher, who was visiting from Graz, Austria.

Dr. Wheeler spent most of April and May 1987 on Study-Research Leave at the Helsinki University of Technology, Helsinki, Finland, interacting with the experts in functional differential and integrodifferential equations at that institution. While in Helsinki he delivered a series of two talks on various aspects of feedback stabilization of viscoelastic rods and beams. He also discussed with Dr. Olof Staffans a problem of feedback stabilization of finite dimensional systems of functional differential equations with infinite delays using some previous joint work with Staffans on the structure of the range and null space of finite dimensional convolution operators. Wheeler also visited the Universität Gesamthochschule Paderborn, West Germany, and discussed problems of energy dissipation in multidimensional viscoelastic structures with Dr. Jan Prüss of that institution.

While in Paderborn, Wheeler delivered a colloquium lecture on the sensitivity of boundary feedback stabilization schemes to small time delays.

In October of 1986, Hannsgen and Wheeler presented talks entitled "Regularity for Integro-Partial Differential Equations" and "Boundary Stabilization of Integro-Partial Differential Equations", respectively, at the Sixth Annual Southeastern-Atlantic Regional Conference on Differential Equations held in Clemson, South Carolina. Wheeler attended the 1986 SIAM National Meeting in Boston in July of 1986. Some of the ideas that came up in conversations with experts at this meeting influenced our work.

Wheeler spent the week of February 29 to March 6, 1988 at Southern Illinois University at Carbondale. He gave a colloquium lecture based on results in [7] and a seminar lecture based on [18] at SIU. While in Carbondale, he also discussed problems of wave propagation in three dimensional viscoelastic solids with Dr. Ronald Grimmer of SIU.

Wheeler spent the period June 14 to July 9, 1988 at Universität Graz in Graz, Austria, continuing the collaboration with Dr. Wolfgang Desch initiated during Desch's Postdoctoral appointment supported by the subject grant at Virginia Tech in January-May, 1987. Wheeler and Desch completed their work on loss of well-posedness [18] during this period. In addition, the collaboration between the Principal Investigators and Dr. Desch on the input-output problem for viscoelastic systems with variable gain feedbacks which to date has resulted in publication [20] was initiated during this period.